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The Decay Modes $\eta^0 \to \gamma + e^+ + e^-$, $\eta^0 \to \gamma + \mu^+ + \mu^-$ and $\eta^0 \to 2\pi + e^+ + e^-$.

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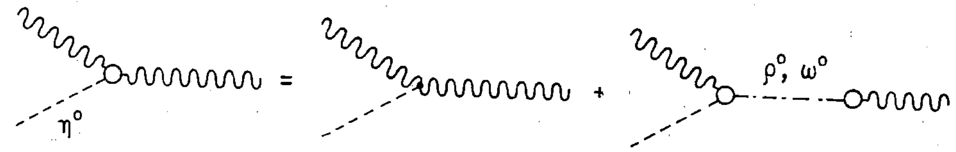
In this note we shortly report on the results of a calculation of the spectra and probabilities of $\eta^0 \to \gamma + e^+ + e^-$ and $\eta^0 \to \gamma + \mu^+ + \mu^-$. We also discuss the decay mode $\eta^0 \to 2\pi + e^+ + e^-$. We assume for η^0 the quantum numbers $J = 0$, $P = -1$, $I = 0$, $C = G = +1$ [1]. We approximate the $\eta^0 \to 2\gamma$ amplitude (with one photon off-mass-shell) with a subtraction term and the pole terms due to ρ^0 and ω^0:

(1)



We call m the invariant mass of the emitted two-lepton system

$$m = [(E_+ + E_-)^2 - (p_+ + p_-)^2]^{\frac{1}{2}}\,, \tag{2}$$

where $E_\pm$ and $p_\pm$ are the energies and momenta of the positive (negative) lepton. The possible values of m are between $2m_l$ (where m_l is the lepton mass) and m_η (η-mass $\div$ 550 MeV). The number of events with m between m and $m + dm$ is given by

$$N(m)\,dm = [\tau(2\gamma)]^{-1}\frac{4\alpha}{3\pi}\frac{1}{m}\left[1-\left(\frac{m}{m_\eta}\right)^2\right]^3\left[1+2\left(\frac{m_l}{m}\right)^2\right]\cdot \left[1-\left(\frac{2m_l}{m}\right)^2\right]^{\frac{1}{2}}\left[v\frac{m_v^2}{m_v^2-m^2}+(1-v)\right]^2 dm\,, \tag{3}$$

where v is a parameter depending on the relative values of the residui of the poles

(1) P. L. Bastien, J. P. Berge, O. I. Dahl, M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld and M. B. Watson: *Phys. Rev. Lett.*, 8, 114 (1962).

and the subtraction constant. In (3) $\tau(2\gamma)$ is the partial lifetime for $\eta^0 \to 2\gamma$. The mass m_v is some average of the ρ^0 and ω^0 mass. The value $v=0$ corresponds to keeping only the subtraction term in the expansion (1) (constant form factor). The value $v=1$ corresponds to keeping only the vector meson poles neglecting the subtraction constant. There exists an experiment on the π^0 form-factor that gives the value of the derivative at the origin of the π^0 form-factor with respect to the squared four-momentum of the off-mass-shell photon [2]. If we assume that η^0 is the eighth member of the unitary symmetry octet containing π and K we can tentatively make use of unitary symmetry to obtain a value for v from the quantity measured in the π^0 experiment. We get $v=-7\pm 5$. Needless to say, such an extrapolation of unitary symmetry arguments to a low-energy region where mass differences are quite important may be completely illusory. In Fig. 1 we have reported the entire spectrum of $\eta^0 \to \gamma+\mu^+ +\mu^-$ and the high-energy tail of $\eta^0 \to \gamma+e^+ +e^-$ in arbitrary units, for values of $v=0$, 1, and -7. The spectra have the right relative normalizations, *i.e.*, apart from the common factor $(4\alpha/3\pi)(\tau(2\gamma))^{-1}$, $N(m)\,dm$ gives, for each case, directly the number of events with m between m and $m+dm$.

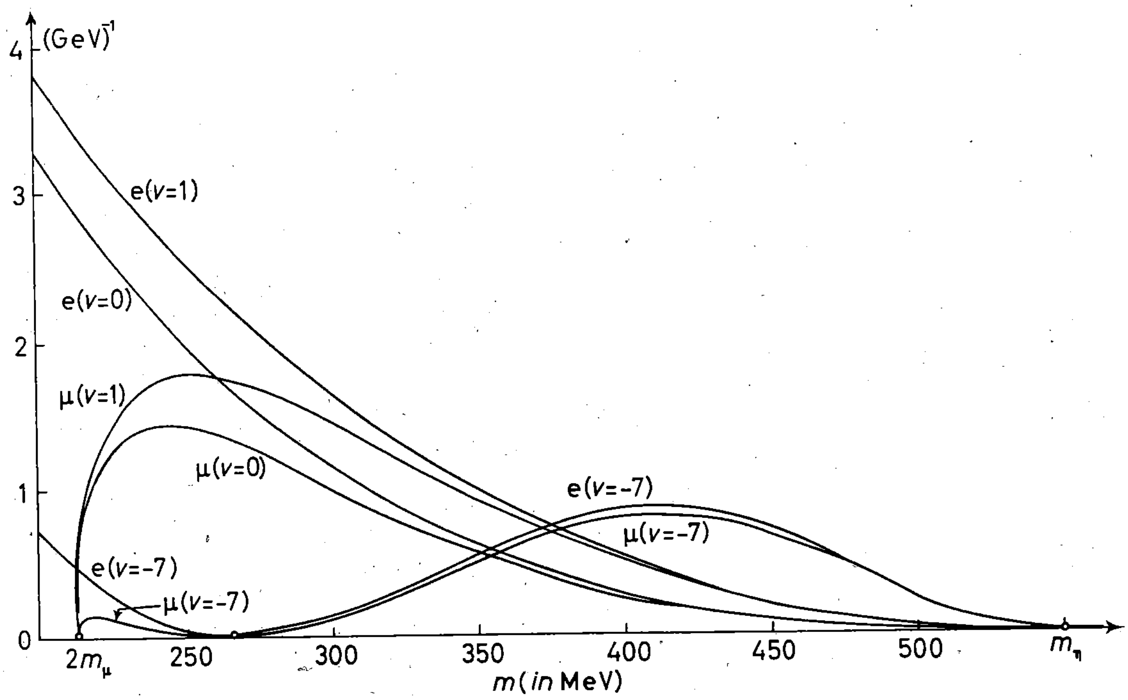


Fig. 1. – Graphs of $(3\pi/4\alpha)\tau(2\gamma)N(m)$, where $N(m)\,dm$ is the number of events with m (invariant lepton mass) between m and $m+dm$, for $\eta^0\to\gamma+e^+ +e^-$ and $\eta^0\to\gamma+\mu^+ +\mu^-$ with different values of the parameter v.

Of course, for $\eta^0\to\gamma+e^+ +e^-$ the spectrum receives its biggest contribution from smaller values of m down to $2m_e$, as given by (3). The branching ratios ρ_e and ρ_μ for $\eta^0\to\gamma+l^+ +l^-$ relative to $\eta^0\to 2\gamma$ can be obtained by integrating (3). For $v=0$

(2) N. P. Samios: *Phys. Rev.*, **121**, 275 (1961).
(3) M. Gell-Mann: *Phys. Rev.*, **125**, 1067 (1962); Y. Neeman: *Nucl. Phys.*, **26**, 222 (1961).

one has

$$(4)\qquad \varrho_\mu=\frac{2\alpha}{3\pi}\left\{\left(-\frac{7}{2}+13r^2+4r^4\right)(1-4r^2)^{\frac12}+2(1-18r^4+8r^6)\lg\frac{1+(1-4r^2)^{\frac12}}{2r}\right\},$$

with $r=m_\mu/m_\eta$. Putting $r=0$ one obtains

$$\varrho_e=\frac{2\alpha}{3\pi}\left[\log\left(\frac{m_\eta}{m_e}\right)^2-\frac{7}{2}\right],$$

which is the well-known Dalitz formula [4].

By numerical integration, with $m_v=750$ MeV, we find

$$(5)\qquad \varrho_e=(16.2+0.47v+0.035v^2)\cdot10^{-3},$$

$$(5')\qquad \varrho_\mu=(55.8+21.9v+2.74v^2)\cdot10^{-5}.$$

From (5) and (5′) we see that ϱ_e is much less sensitive to v than ϱ_μ and, taking the model literally, we expect, independently of v, $\varrho_e>14.6\cdot10^{-3}$ and $\varrho_\mu>12.1\cdot10^{-5}$. With $v=0$ $\varrho_e=16.2\cdot10^{-3}$ and $\varrho_\mu=55.8\cdot10^{-5}$. With $v=1$ $\varrho_e=16.7\cdot10^{-3}$ and $\varrho_\mu=80.4\cdot10^{-5}$. With $v=-7$ (unitary symmetry extrapolation) $\varrho_e=14.6\cdot10^{-3}$ and $\varrho_\mu=36.9\cdot10^{-5}$. Note that ϱ_e alone would determine v (or better two possible values for v) and then one would be able to predict ϱ_μ and the shapes of the spectra. A reaction such as $\gamma+p\to\eta^0+p$ followed by $\eta^0\to\mu^++\mu^-+\gamma$ simulates $\gamma+p\to\to\mu^++\mu^-+\gamma+p$. With a cross-section $\sim10^{-30}$ cm² for $\gamma+p\to\eta^0+p$ the apparent cross-section for $\gamma+p\to\mu^++\mu^-+\gamma+p$ would then be at least $\sim10^{-34}$ cm².

The decay mode $\eta^0\to\pi^0+e^++e^-$ is forbidden, at lowest electromagnetic order, by charge conjugation invariance. Similarly $\eta^0\to\pi^0+\pi^0+e^++e^-$ only occurs at higher electromagnetic order. The mode $\eta^0\to\pi^++\pi^-+e^++e^-$ occurs at lowest electromagnetic order from the internal conversion of the photon emitted in $\eta^0\to\pi^++\pi^-+\gamma$. For $\eta^0\to\pi^++\pi^-+\gamma$ the general form of the matrix element is

$$(6)\qquad \frac{1}{(2\pi)^2}\frac{\delta(P-p^{(+)}-p^{(-)}-k)}{\sqrt{16EE_+E_-k}}H\varepsilon_{\mu\nu\lambda\varrho}\varepsilon_\mu p_\nu^{(+)}p_\lambda^{(-)}P_\varrho,$$

where P, $p^{(+)}$, $p^{(-)}$ and k are the momenta of η^0, π^+, π^- and γ; E, E^+, E^-, are their energies, ε_μ is the photon polarization vector, and H is a form factor, that we shall take as a constant. The pion distribution from (6) is (in c.m.)

$$(7)\qquad \frac{|H|^2m_\eta}{(4\pi)^3}\{(m_\eta^2+m_\pi^2-2m_\eta E_-)E_+^2-[2m_\eta E_-^2-(3m_\eta^2+2m_\pi^2)E_-+$$
$$+(m_\eta^2+2m_\pi^2)m_\eta]E_++(m_\eta^2+m_\pi^2)E_-^2-(m_\eta^2+2m_\pi^2)m_\eta E_-+(\tfrac14m_\eta^2+m_\pi^2)m_\eta^2\}\,dE_+dE_-.$$

In Fig. 2 we have reported the pion spectrum (dw/dE_π) from $\eta^0\to\pi^++\pi^-+\gamma$ obtained

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(4) R. H. DALITZ: *Proc. Phys. Soc.*, A **64**, 667 (1951); N. KROLL and M. WADA: *Phys. Rev.*, **98**, 1355 (1955).

by integrating eq. (7). The quantity reported is $|H|^{-2}m_\eta^{-7}(\mathrm{d}w/\mathrm{d}E_\pi)\cdot 10^7$. The total rate is

$$w(\eta^0\to\pi^+\pi^-\gamma) = 3.92\cdot 10^{-8}\,|H|^2 m_\eta^7\,. \tag{8}$$

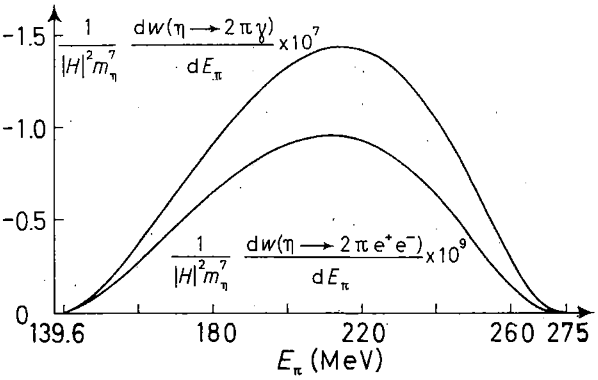

Fig. 2. – Pion spectra from $\eta^0\to\pi^++\pi^-+\gamma$ and from $\eta^0\to\pi^++\pi^-+e^++e^-$, under the assumption of a constant form factor H.

The spectra and probability for $\eta^0\to\pi^++\pi^-+e^++e^-$ can be obtained from (6) by internal conversion of the virtual photon. Defining

$$L_\mu = \frac{1}{(2\pi)^{\frac{7}{2}}}\frac{\delta(P-p^{(+)}-p^{(-)}-k)}{\sqrt{8EE_+E_-}}H\varepsilon_{\mu\nu\lambda\varrho}p_\nu^{(+)}p_\lambda^{(-)}P_\varrho\frac{1}{k^2}\,, \tag{9}$$

we obtain for the total rate

$$w = -\frac{(2\pi)^2}{3}e^2\int \mathrm{d}^3p^{(-)}\,\mathrm{d}^3p^{(+)}k^2L^2\,. \tag{10}$$

The pion spectrum $(\mathrm{d}w/\mathrm{d}E_\pi)$ from $\eta^0\to\pi^++\pi^-+e^++e^-$ is given in Fig. 2. The quantity reported there is $|H|^{-2}m_\eta^{-7}(\mathrm{d}w/\mathrm{d}E_\pi)\cdot 10^9$. One sees that the shape of the spectrum is quite similar to that for $\eta^0\to 2\pi+\gamma$. The total rate is

$$w(\eta^0\to\pi^+\pi^-e^+e^-) = 2.59\cdot 10^{-10}\,|H|^2 m_\eta^7\,. \tag{9}$$

From (9) and (8) we find for the branching ratio $(\eta^0\to\pi^+\pi^-e^+e^-)/(\eta^0\to\pi^+\pi^-\gamma)$ a value of 0.0066. This result should be essentially independent of the assumptions made.

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